

SUN/213.4

Starlink Project
Starlink User Note 213.4

G. Privett, T. Jenness, H. Matthews, V. Barnard

10 May 2021

FLUXES
JCMT Position and Flux Density
Calibration
Version 1.2
User's Manual

Abstract

FLUXES calculates approximate **topocentric positions** of the planets and also **integrated flux densities** of five of them at several wavelengths. These provide calibration information at the effective frequencies and beam-sizes employed by receivers on the JCMT telescope based on Mauna Kea, Hawaii.

The FLUXNOW command, runs FLUXES for the current time and date. This allows you to obtain planetary positions and flux levels quickly.

Contents

1	Introduction	1
2	Frequencies, brightness and beam-width	1
3	Using FLUXES	2
4	Planetary brightness and flux density calculations	4
5	Using FLUXNOW	6
6	Acknowledgements	6
A	Running in batch mode	7
B	How does FLUXES work?	8
	B.1 Calculating topocentric and geocentric positions and distances	8
	B.2 Calculating the solid angle subtended by the planet	10
	B.3 Calculating Mars' brightness temperatures	11
	B.4 Converting brightness temperatures to flux densities	12

List of Figures

1	Martian brightness temperatures	5
2	Simplified geometry for planetary calculations	9

1 Introduction

FLUXES calculates approximate **topocentric positions** of the planets and also **integrated flux densities** of five of them at a number of wavelengths for the JCMT telescope on Mauna Kea, Hawaii. These provide calibration information at the effective frequencies and beam-sizes employed by receivers on this telescope.

The filter centre and widths are shown for each wavelength used. The filter frequencies are the effective frequencies for 1mm of water vapour.

The calculated planet positions should normally be accurate to better than 1 arcminute. The value for the Moon is less accurate than this, and for critical applications requiring sub arc-second accuracy you should use a different method.

The planet positions are estimated using the SLALIB library. See SUN/67 for more details on the SLA_PLANET library function.

The FLUXNOW command, runs the program for the current time and date. This gives you current planetary positions and flux levels for all the planets.

2 Frequencies, brightness and beam-width

The effective frequencies and half-power beam-widths for the filters, and the brightness temperatures for the planets (except Mars) are read from the plain text files:

ukt14_old.dat – for dates before 7-Aug-92.

ukt14.dat – for the period 7-Aug-92 to 23-May-96.

scuba.dat – for dates after 23-May-96.

scuba2.dat – for dates after 1-Jan-2007.

The information in the files was supplied by the observatory. The differences mainly reflect changes in detector/telescope hardware.

The program selects the appropriate .dat file to use, based on the date you specify. The calculated temperature has an rms error of about 0.13K (negligible).

Alternatively, if the ADAM parameter "FILTER" is set to CUSTOM on the command line, then the parameters of a single filter to use are obtained from the user using a set of environment parameters, rather than being read from a file as described above. The planetary brightness temperature can either be supplied explicitly or can be looked up from a specified NDF using the supplied frequency as the index (see parameter BTEMP below). For the case of Uranus, the FLUXES package includes two files that can be used for this purpose, both produced as part of the calibration effort for Herschel SPIRE spectroscopy:

`esa2_uranus.sdf`: The initial Uranus model at Herschel launch.

`esa4_uranus.sdf`: An updated model that includes Spitzer data for constraining the continuum, but no lines.

See Herschel Planetary Calibrator Models release notes for more information on these models.

The following parameters are used to describe the custom filter:

FREQ: The central frequency of the filter in GHz.

NB: The number of Gaussian components to the beam - must be 1 or 2.

HPBW1: Half-power beam width of the first Gaussian component of the beam, in arcsec.

HPBW2: Half-power beam width of the second Gaussian component of the beam, in arcsec. Unused if NB is 1.

AMP1: Amplitude of the first Gaussian component of the beam, in arcsec, normalised so that AMP1 plus AMP2 is one. Unused if NB is 1.

AMP2: Amplitude of the second Gaussian component of the beam, in arcsec, normalised so that AMP1 plus AMP2 is one. Unused if NB is 1.

BTEMP: Either the numerical planetary brightness temperature at the required frequency, in K, or the name of a 1-dimensional NDF. If an NDF name is given, it should contain the required brightness temperature as a function of frequency. The current WCS Frame should contain a spectral axis giving frequency in units of GHz. The NDF is used to look up the required temperature at the frequency specified by parameter FREQ. Note, this parameter is not used if Mars is the selected planet (see appendix B.3 for a description of the brightness temperature used for Mars).

NOTE: A group of lines of explanatory text to display. Defaults to none.

Bear in mind that Saturn can only be used for calibration when the rings are not open, and the Martian brightness temperatures are probably only accurate to around $\pm 5\%$.

3 Using FLUXES

To run FLUXES or FLUXNOW, type its name (in lower case) after the prompt:

```
% fluxes
```

The program first asks you what output you want to generate:

```
JCMT FLUXES
```

```
POS - Do you require planetary data? (Y/N) /YES/ > Y
```

```
FLU - Do you require flux data? (Y/N) /YES/ > Y
```

In the above example I opted to see both sets of output data. If you were only interested in the planetary positions, reply “N” to the second prompt. (Prompts asking for a Y/N answer can also be answered using true/false or t/f or y/n – case is not significant.)

Next, indicate where you want the information displayed or stored:

```
SCREEN - Display output to terminal? (Y/N) /YES/ > Y
OFL - Store output in a file? (Y/N) /NO/ > Y
OUTFILE - Name of output file /'fluxes.dat'/ >
```

```
Data will be written to file: /home/user1/gjp/flux/fluxes.dat
```

If you reply “N” to the OFL prompt, the next lines shown won’t appear.

Next, you are asked to specify the time and date for which the information should be derived. If you set the value of NOW to Y, the current date and time are automatically obtained from your computer. Otherwise, you must specify the time and date yourself, as appears below:

```
NOW - Use todays date/time? (Y/N) /YES/ > N
```

```
DATE - UT DATE (DD MM YY) /'12 09 96'/ > 16 09 96
```

```
TIME - UT TIME (HH MM SS) /'12 00 00'/ > 08 45 00
```

Since you might only require flux and positional information for one planet, you are given the option of selecting it by specifying a value for the parameter PLANET. When prompted, give the name of any planet, the Sun, the Moon, or ‘ALL’ (meaning all the planets plus the Sun and Moon). The information requested is then output to the screen and/or to the file (depending on the values you gave the parameters SCREEN and OFL). An example of the output generated is:

```
Planetary Submillimetre Fluxes for the JCMT
```

```
UT: 11:25:55 UT Date:18-Sep-1996 HST: 1:25:55
```

```
LST: 00:54:39.1699 MJD (TT): 50344.477 Epoch: 1996.7145
```

Body	RA (h m s)	TRIMRA (arcsec/sec)	Dec (d m s)	TRIMDEC (arcsec/sec)	GD (au)	AMASS
SUN	11 44 40.1798	0.0379	+ 1 39 33.4379	-0.0161	1.004691	13.375
MERCURY	11 33 12.9974	-0.0334	- 0 15 11.5562	0.0277	0.652112	13.375
VENUS	8 57 54.4433	0.0475	+16 20 19.3720	-0.0096	0.931404	13.375
MARS	8 32 2.6334	0.0268	+19 56 37.9464	-0.0057	2.022026	13.375
JUPITER	18 35 36.7652	0.0021	-23 23 39.5939	0.0001	4.866478	13.375
SATURN	0 21 4.4782	-0.0030	- 0 33 58.9245	-0.0013	8.508262	1.078
URANUS	20 12 46.1430	-0.0008	-20 33 30.0554	-0.0001	19.190398	5.546
NEPTUNE	19 47 41.0331	-0.0004	-20 39 31.4554	-0.0001	29.655142	10.659
PLUTO	16 4 56.6035	0.0008	- 7 52 56.8739	-0.0005	30.327774	13.375
MOON	15 54 23.9864	0.7379	-16 17 3.6688	-0.0371	0.002580	13.375

```
MARS
```

```
South pole is Earth-facing; Inclination Angle = 73.30 degrees
```

```
Semi-diameter = 2.31 arcsecs Solid angle = 3.95E-10 sterads
```

Filter Wavel. micron	Centre Freq (GHz)	Filter Width (GHz)	Total Flux (Jy)	Flux in beam (Jy)	Brightness Temperature (K)	HPBW assumed (arcsecs)
2000	146.0	39.0	52.15	51.82	205.1 +- 0.0	33.8
1300	221.0	60.0	119.63	117.86	207.1 +- 0.0	22.3
1100	264.0	60.0	170.57	167.01	208.0 +- 0.0	18.7
850	350.0	30.0	298.81	287.79	209.3 +- 0.0	14.0
750	407.0	30.0	402.85	383.13	210.0 +- 0.0	12.1
600	483.0	51.0	564.68	526.30	210.9 +- 0.0	10.2
450	677.0	30.0	1093.62	954.69	212.5 +- 0.0	7.3
350	866.0	30.0	1761.30	1414.23	213.7 +- 0.0	5.7
200	1490.0	72.0	4915.53	3946.91	216.3 +- 0.0	5.7

and so on....

If you encounter problems, please ask your system manager to consult Appendix A.

4 Planetary brightness and flux density calculations

The calibration of the temperature and flux density scales for the JCMT antenna ultimately rests on accurate knowledge of the brightness temperatures of the planets as a function of frequency. The main planets used are: Mars, Jupiter, Saturn, Uranus, and Neptune. Brightness and flux values are not provided for Mercury, Venus, or Pluto/Charon. Mars is used as the primary calibrator; that is, the brightness temperatures of all other planets are referred to the brightness temperature of Mars. Since Mars is a solid body, it is relatively secure to use a model of its brightness temperature as a function of date. This information can then be used to calibrate observations of the other planets.

Once the brightness temperatures are known, it becomes possible to calculate the flux density received at the Earth from a given planet, corrected for the apparent size of the planet and the beam size of the JCMT at the frequency being used. This is done by FLUXES and FLUXNOW.

Historically, FLUXES was developed for use with the bolometer UKT14. For each of the available UKT14 filters the brightness temperature and total flux density of the planet is calculated, along with the flux density visible to a 65mm aperture (the standard detector width) – shown in the tables under the heading Flux in beam. The latter is calculated from the beam width given in the final column – if you have reason to believe that the beam width is different, or you have used a different aperture, you will have to re-do the calculation.

The planetary brightness temperatures used are obtained from the measurements of Griffin et al. (1986, *Icarus*, 65, 244), Orton et al. (1986, *Icarus*, 67, 289), and Griffin & Orton (1993, *Icarus*, 105, 537). The errors in brightness temperatures are the rms internal errors of the observations.

From MJD 46040 until MJD 50000 (*i.e.* JD 2450000.5, corresponding to 1995 October 10), FLUXES uses $\lambda 350\mu\text{m}$ Martian brightness temperatures T_b derived from the model described by Wright (1976, *Ap.J.*, 210, 250). This uses an adjusted model of the thermal behaviour of Mars' surface and geometric factors to predict the flux received at the Earth and the whole-disk brightness temperature at 40-day intervals. The model does not include the effects of dust storms and the Martian polar cap. At all other times, the model has a maximum uncertainty of within $\pm 5\%$

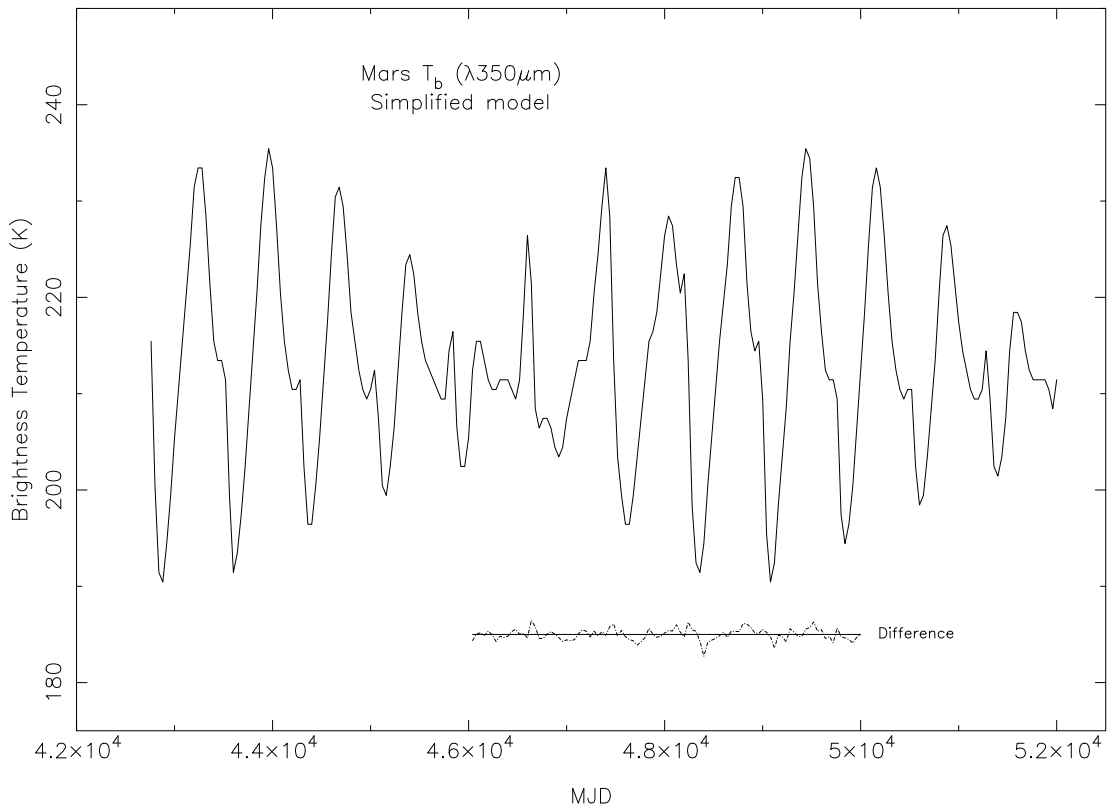


Figure 1: Martian brightness temperatures

. Since the results for Mars are derived from models, no errors are quoted in the output of FLUXES.

Before MJD 46040 and, more importantly, since MJD 50000, FLUXES uses a modification of Wright’s rotating cratered asteroid program (Wright, E.L., 2007 arXiv:astro-ph/0703640). This reproduces 82% of the T_b values in Table 6 of Wright (1976) exactly (where they are rounded to the nearest Kelvin), while the other 18% differ by only ± 1 K. A low-precision formula by Van Flandern and Pulkinnen, (1979, *Ap.J.Supp.*, 41, 391) was used to compute the geometric factors (Mars-Sun distance R , Martian declinations of the Sun and the Earth D_S and D_E , and the difference in Martian right ascensions between the Earth and the Sun $A_S - A_E$) far into the future.

The values of T_b for Mars are calculated for the entire date range at intervals of 40 days. FLUXES interpolates between the nearest two values to obtain the value for the requested date and time. Figure 1 shows the variation of disk-averaged Martian $\lambda 350\text{-}\mu\text{m}$ brightness temperatures over a period of 25.2 years, beginning in late 1975, calculated using a rotating cratered asteroid model, and corrected for the microwave background. The differences between this model and the full model by Wright (1976) over the range for which data is available is shown dotted below, where the solid horizontal line is 0 K T_b difference.

Over the interval for which both the original Wright model and the simplified model are valid, the differences in brightness temperatures have an rms error of 0.13 K (*i.e.* negligible) after

removing a constant offset of -2.56 K, assumed due to the microwave background.

5 Using FLUXNOW

FLUXNOW is a script that calls the program FLUXES in such a way that the results for the current date and time are returned to you without further input. For your convenience, the information generated is placed in a file called `fluxes.dat` in your current directory. If a file of that name already exists, the information is appended to the existing file. Note that, as with FLUXES, the FLUXNOW command which runs the script should be typed at the UNIX prompt in lower case, as given below:

```
% fluxnow
```

6 Acknowledgements

FLUXES was originally developed as part of the 'JCMT Utilities' package maintained by the JCMT Software Group at Hilo, Hawaii. It is loosely based on code originally developed by B.D. Yallop and C.Y. Hohenkerk of HM Nautical Almanac Office.

A Running in batch mode

Occasionally it would be useful to retrieve planet fluxes without having to run FLUXES interactively. The current version has been modified so that in certain circumstances the flux values can be written to ADAM parameters and retrieved (this is somewhat easier than parsing the output text file).

When FLUXES is run with a single planet name and filter the calculated flux values for that combination are written to the following ADAM parameters:

HPBW: Half-power beam width used to calculate F_BEAM (arcsec)

F_CENTRE: the centre frequency of the selected filter (GHz)

F_WIDTH: the filter width (GHz)

F_TOTAL: the total flux (Jy)

F_BEAM: the total flux in the beam (Jy)

T_BRIGHT: the brightness temperature of the planet (K)

T_ERROR: the error in the brightness temperature (K)

SEMI_DIAM: Semi-diameter of the selected planet (arcsec). If 'ALL' planets are selected then the parameter will retain the value of the last planet processed (Neptune)

SOLID_ANG: Solid angle of the selected planet (steradians). If 'ALL' planets are selected then the parameter will retain the value of the last planet processed (Neptune)

If FLUXES is run from the shell (as opposed to running it as a monolith (see later in the section for information on running fluxes as a monolith)) the parameter values can be retrieved by using either the KAPPA parget command or, if running from perl, the par_get routine supplied as part of the NDFPERL package.

Here is an example, running FLUXES from the command-line with no output except via the parameter system

```
% fluxes pos=no flu=yes screen=no ofl=no now filter=850 apass=no \  
    planet=uranus msg_filter=quiet  
% parget f_total fluxes  
72.4569
```

Note how MSG_FILTER was used to turn off default messages, and that the FILTER and PLANET were selected explicitly. Also, useful output was obtained even though fluxes was instructed to write to neither the screen or a file (and MSG_FILTER=QUIET hides output even if SCREEN=yes).

It is possible to run FLUXES as a monolith but this requires that the fluxes monolith is run directly (not via the fluxes.sh shell script), that the environment variables FLUXES_DIR and FLUXPWD are set to the location of the fluxes binary and the current working directory respectively. If it is running as a monolith (specifically as an I-task) the parameter values can then be retrieved from the monolith (e.g. by using STARTCL).

B How does FLUXES work?

In this section we will discuss the actual calculations performed by FLUXES to produce its position and flux output. This depends somewhat upon the planet for which information is requested. For Mars, the following steps are performed:

- Topocentric and Geocentric R.A. and Dec positions, distances and airmass are calculated
- The solid angle subtended by the planet is calculated
- The brightness temperature at $350\mu\text{m}$ (857 GHz), $T_{b_{857}}$, is calculated by linear interpolation from 40-day interval values, from either Wright's original model or the rotating cratered asteroid model, depending upon the date (see section 4)
- The brightness temperature at 3.3mm (90 GHz), $T_{b_{90}}$, is calculated following the simple relation given by Ulich (1981, *A.J.*, 86, 1619)
- A logarithmic interpolation between $T_{b_{857}}$ and $T_{b_{90}}$ calculates the brightness temperature for each filter wavelength
- The brightness temperature is converted to integrated and beam-corrected flux densities.

Alternatively, for Jupiter, Saturn, Neptune and Uranus, the following calculations are performed:

- Topocentric and Geocentric R.A. and Dec positions and airmass are calculated
- The solid angle subtended by the planet is calculated
- The brightness temperature for the relevant planet(s) for each filter is given by a look-up table as described in section 2
- The brightness temperature is converted to integrated and beam-corrected flux densities.

Finally, for Mercury, Venus, the Sun and the Moon, only the first step is performed:

- Topocentric R.A. and Dec positions, distances and airmass are calculated

Below we will discuss the individual calculations in more depth.

B.1 Calculating topocentric and geocentric positions and distances

The geometry for the calculations of positions and distances to the requested planet is shown in Figure 2.

The topocentric vector of a planet, V_t , or the geocentric vector of the planet, V_g , are calculated with reference to the geocentric vector of the Sun, VSG , the heliocentric vector of the planet, VSP , and the geocentric vector of the observer, VGO . Also used by the current version of FLUXES are the geocentric vector to the Moon, VGM , the vector from the Sun to the Earth-Moon barycentre,

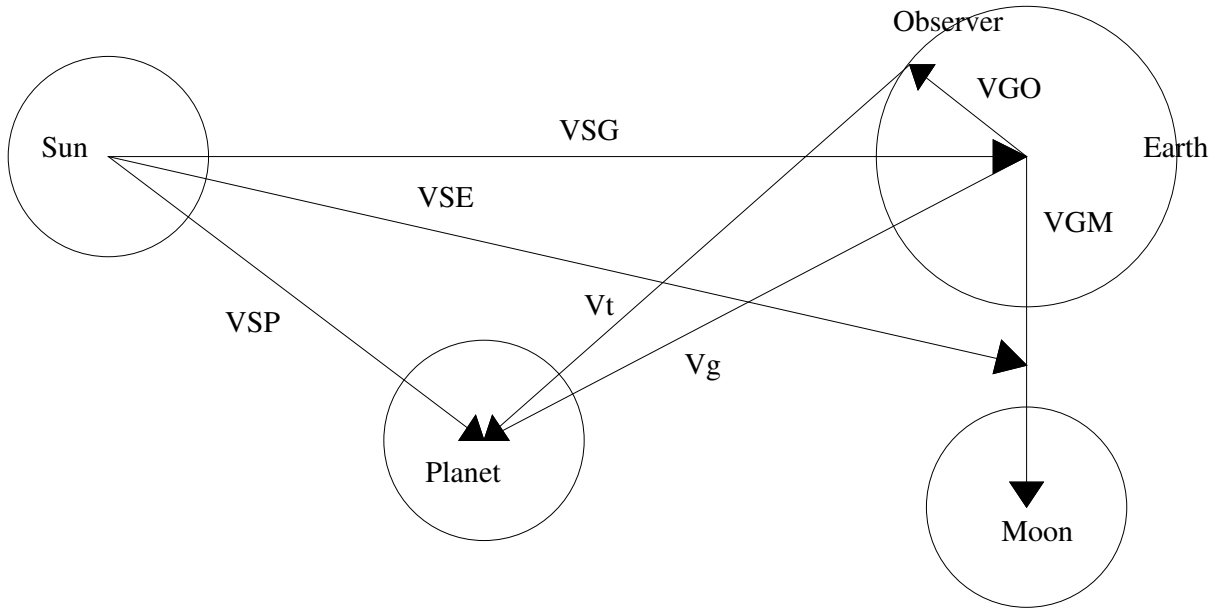


Figure 2: Simplified geometry for planetary calculations

VSE , and the knowledge that the geocentric vector of the Earth-Moon barycentre is given by $(0.012150581 \times VGM)$.

When the directions of these vectors are as shown in Figure 2, the geocentric position and velocity vector of the planet is given by

$$V_g = -VSG + VSP$$

where the geocentric vector of the Sun, VSG , may be given as

$$VSG = VSE - 0.01215081(VGM).$$

Similarly, the topocentric position and velocity vector of the planet is given by

$$\begin{aligned} V_t &= V_g - VGO \\ &= VSP - VSG - VGO \\ &= VSP - (VSE - 0.012150581(VGM)) - VGO \end{aligned}$$

All these vector quantities must be precessed to the appropriate date. The prime symbol (\prime) will be used in the following equations to indicate precessed positions.

Once the necessary vector is known, the actual distance is calculated as the square root of the summed square of each component, e.g.,

$$d_g = \sqrt{V'_{gx}{}^2 + V'_{gy}{}^2 + V'_{gz}{}^2}$$

and hence the light travel time to the planet is given by

$$T_l = \frac{d_g}{c}$$

where c is in the appropriate units. The position vector of the planet as viewed from the Earth should then be corrected for planetary aberration, e.g.

$$V'_{g_x,abb.} = V'_{g_x} - (T_l \times V'_{g_x})$$

where V'_{g_x} is the (uncorrected) velocity vector component in the x direction. The corrected position vector $V'_{g,abb.}$ or $V'_{t,abb.}$ can then be converted into standard R.A. and Dec coordinates, and the airmass can be calculated from the topocentric coordinates.

B.2 Calculating the solid angle subtended by the planet

Several calculations in this section make use of two references in particular; planetary information is mainly taken from the “Report of the IAU working group on cartographic coordinates and rotational elements of the planets and satellites”, which is periodically updated and published online and in *Celestial Mechanics and Dynamical Astronomy*. Precession formulae come from Lieske (1979, *A & A*, 73, 282).

To calculate the solid angle of the planet in question it is necessary to take account of the ellipticity of the planet, via the direction of its North Pole. The coordinates of the pole, RA_{pole} and Dec_{pole} are calculated directly from the IAU report formulae for the date given and then precessed according to Lieske’s spherical coordinate formulae. If using J2000.0 for the polar coordinates, the precession parameters ζ_A , z_A and θ_A (see Lieske’s Fig. 1) are defined in arcseconds as

$$\zeta_A = 2306.2181t + 0.30188t^2 + 0.017998t^3$$

$$z_A = 2306.2181t + 1.09468t^2 + 0.018203t^3$$

$$\theta_A = 2004.3109t - 0.42665t^2 - 0.041833t^3$$

and the following quantities can be defined

$$C = -(\sin Dec_{pole} \sin \theta_A) + (\cos Dec_{pole} \cos \theta_A \cos(\zeta_a + RA_{pole}))$$

$$S = \cos Dec_{pole} \sin(\zeta_a + RA_{pole})$$

$$D = (\sin Dec_{pole} \cos \theta_A) + (\cos Dec_{pole} \sin \theta_A \cos(\zeta_a + RA_{pole}))$$

The precessed coordinates, RA'_{pole} and Dec'_{pole} are then calculated as

$$RA'_{pole} = z_A + \arctan(S, C)$$

$$Dec'_{pole} = \arcsin(D)$$

$$= \arccos(\sqrt{S^2 + C^2})$$

where (S, C) is the complex number formed from the quantities S and C . From these precessed coordinates, the planetocentric declination of the Earth can be defined as

$$Dec'_{Earth} = \arcsin(-\sin Dec'_{pole} \sin Dec'_g - \cos Dec'_{pole} \cos Dec'_g \cos(RA'_{pole} - RA'_g))$$

where RA'_g and Dec'_g are the precessed geocentric coordinates of the planet. The polar inclination angle of the planet, α_{pole} , is then given by

$$\alpha_{pole} = Dec'_{Earth} + \frac{\pi}{2}$$

or, if $Dec_{Earth} > 0$ so the South Pole is facing Earth,

$$\alpha_{pole} = \frac{\pi}{2} - Dec'_{Earth}.$$

With this calculation performed the semi-major axis of the planet as seen from Earth, $R_{semi-major}$ is given as

$$R_{semi-major} = \frac{R_p(1 - \epsilon_p)}{1 - \epsilon_p \cos \alpha_{pole}}$$

where R_p is the planet's radius and ϵ_p is its ellipticity. Hence its geometrical mean radius R_{gm} is

$$R_{gm} = \sqrt{R_p \times R_{semi-major}}$$

and its semi-diameter as seen from the Earth, D_{semi} is

$$D_{semi} = \frac{R_{gm}}{d_g}$$

where d_g is the geocentric distance of the planet from the Earth as calculated in section B.1. Finally, the solid angle subtended by the planet from the Earth, Ω_p , is

$$\Omega_p = \pi D_{semi}^2$$

B.3 Calculating Mars' brightness temperatures

For Mars, the brightness temperature at each filter wavelength is calculated via a logarithmic interpolation between brightness temperatures at $350\mu\text{m}$ (857 GHz), $T_{b_{857}}$, and another at 3.3mm (90 GHz), $T_{b_{90}}$. The first of these is generated from one of two models both from Wright's work, the results of which are held as an array in FLUXES. In the array, values for $T_{b_{857}}$ are given at 40-day intervals, and so a linear interpolation is performed to ascertain a value for the given date.

The 3.3mm point is calculated from the relation given by Ulich (1981, *A.J.*, 86, 1619):

$$T_{b_{90}} = \frac{206.8}{\sqrt{1.524/d_{hel_{Mars}}}}$$

where 1.524 AU is the mean heliocentric distance of Mars and $d_{hel_{Mars}}$ is the heliocentric distance of Mars for the given date, which can be calculated from the precessed geocentric positions and distances of the Sun ($RA'_{g_{sun}}, Dec'_{g_{sun}}, d_{g_{sun}}$) and Mars ($RA'_{g_{Mars}}, Dec'_{g_{Mars}}, d_{g_{Mars}}$) by defining the following quantity:

$$\cos E = \sin Dec'_{g_{Mars}} \sin Dec'_{g_{sun}} + \cos Dec'_{g_{sun}} \cos Dec'_{g_{Mars}} \cos(RA'_{g_{sun}} - RA'_{g_{Mars}})$$

so that

$$d_{hel_{Mars}} = \sqrt{RA'^2_{g_{sun}} + RA'^2_{g_{Mars}} - 2(\cos E)RA'_{g_{sun}} RA'_{g_{Mars}}}.$$

Thus the brightness temperature at a given frequency, ν GHz, is given by a logarithmic interpolation:

$$T_{b_{\nu}} = T_{b_{90}} + (T_{b_{857}} - T_{b_{90}}) \frac{\ln(\nu - 90)}{\ln(857 - 90)}$$

B.4 Converting brightness temperatures to flux densities

Once the brightness temperature is known for each filter frequency, the integrated flux density of the planet in question, $S_{int_{\nu}}$ is calculated as

$$S_{int_{\nu}} = \frac{2h\nu^3}{c^2} \frac{\Omega_p}{\exp\left(\frac{h\nu}{kT_{b_{\nu}}}\right) - 1}$$

Then, with the assumptions that the planet is a flat disk with constant flux distribution across its disk, and that the beam of the telescope is a Gaussian of HPBW $\theta_{beam_{\nu}}$, the beam-corrected flux density of the planet at Earth, $S_{beam_{\nu}}$ is given by

$$S_{beam_{\nu}} = 1.133 \theta_{beam_{\nu}}^{2/\Omega_p} \left[1 - \exp\left(\frac{1}{1.133 \theta_{beam_{\nu}}^{2/\Omega_p}}\right)\right] S_{int_{\nu}}.$$

The values of the HPBW, $\theta_{beam_{\nu}}$, are supplied in the same look-up file used for the other planets' brightness temperatures.

The implication of the flat-disk assumption for the planet is that the observed HPBW, $\theta_{obs_{\nu}}$, is Gaussian and given by

$$\theta_{obs_{\nu}}^2 = \theta_{beam_{\nu}}^2 + \frac{\ln 2}{2} (2D_{semi})^2$$

for $2D_{semi} < \theta_{beam_{\nu}}$, as discussed originally by Baars (1973, *IEEE trans. on Antennas and Propagation*, 21, 461).